

Information for Examinees Sitting for the Fundamentals of Surveying Examination

updated July 2, 2007

The Fundamentals of Surveying (FS) examination is a **closed-book examination**. Therefore, **no reference material is allowed** in the exam room.

The formulas and information here are provided in both the morning and the afternoon exam booklets. This information is not intended to be an all-inclusive compilation of formulas required for the FS examination. Basic theories, conversions, formulas, and definitions that examinees are expected to know have not been included. As required, special material not included here will be provided in the question statement itself to assist the examinee in solving the question.

In no event shall NCEES be liable for not providing reference material to support all the questions in the FS examination. NCEES reserves the right to revise and update the information contained here as it deems appropriate without informing interested parties. Each NCEES FS examination will be administered using the latest version of this information.

Revision made July 2, 2007: The heading PHYSICS was added to the top of the right-hand column on p. 8. The PHYSICS section on the bottom of p. 10 was moved to the bottom of the right-hand column on p. 8.

CONVERSIONS AND OTHER USEFUL RELATIONSHIPS

$$* 1 \text{ U.S. survey foot} = \frac{12}{39.37} \text{ m}$$

$$* 1 \text{ international foot} = 0.3048 \text{ m}$$

$$* 1 \text{ in.} = 25.4 \text{ mm (international)}$$

$$1 \text{ mile} = 1.60935 \text{ km}$$

$$* 1 \text{ acre} = 43,560 \text{ ft}^2 = 10 \text{ square chains}$$

$$* 1 \text{ ha} = 10,000 \text{ m}^2 = 2.47104 \text{ acres}$$

$$* 1 \text{ rad} = \frac{180^\circ}{\pi}$$

$$1 \text{ kg} = 2.2046 \text{ lb}$$

$$1 \text{ L} = 0.2624 \text{ gal}$$

$$1 \text{ ft}^3 = 7.481 \text{ gal}$$

$$1 \text{ gal of water weighs } 8.34 \text{ lb}$$

$$1 \text{ ft}^3 \text{ of water weighs } 62.4 \text{ lb}$$

$$1 \text{ atm} = 29.92 \text{ in. Hg} = 14.696 \text{ psi}$$

$$\text{Gravity acceleration (g)} = 9.807 \text{ m/s}^2 = 32.174 \text{ ft/sec}^2$$

$$\text{Speed of light in a vacuum (c)} = 299,792,458 \text{ m/s} = 186,282 \text{ miles/sec}$$

$$^\circ\text{C} = (^\circ\text{F} - 32)/1.8$$

$$1 \text{ min of latitude } (\phi) \cong 1 \text{ nautical mile}$$

$$1 \text{ nautical mile} = 6,076 \text{ ft}$$

$$\text{Mean radius of the earth} \cong 20,906,000 \text{ ft} \cong 6,372,000 \text{ m}$$

* Denotes exact value. All others correct to figures shown.

| METRIC PREFIXES | | |
|-----------------------------|--------------|-------------------------|
| Multiple | Prefix | Symbol |
| 10^{-18} | atto | a |
| 10^{-15} | femto | f |
| 10^{-12} | pico | p |
| 10^{-9} | nano | n |
| 10^{-6} | micro | μ |
| 10^{-3} | milli | m |
| 10^{-2} | centi | c |
| 10^{-1} | deci | d |

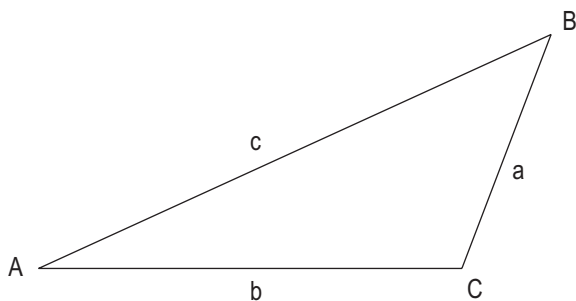
| METRIC PREFIXES | | |
|-----------------------------|--------------|----------|
| Multiple | Prefix | Symbol |
| 10^1 | deka | da |
| 10^2 | hecto | h |
| 10^3 | kilo | k |
| 10^6 | mega | M |
| 10^9 | giga | G |
| 10^{12} | tera | T |
| 10^{15} | peta | P |
| 10^{18} | exa | E |

QUADRATIC EQUATION

$$ax^2 + bx + c = 0$$

$$\text{Roots} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

OBLIQUE TRIANGLES



Law of sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Law of cosines

$$a^2 = b^2 + c^2 - 2bc \cos A$$

or

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

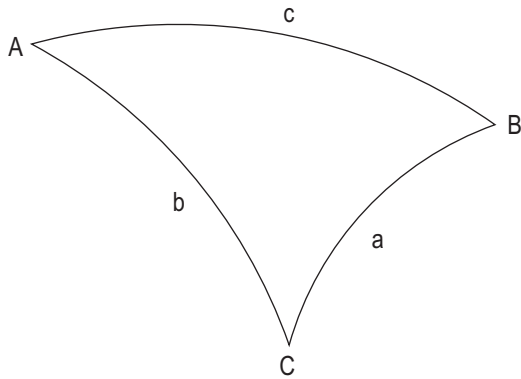
$$\text{Area} = \frac{ab \sin C}{2}$$

$$\text{Area} = \frac{a^2 \sin B \sin C}{2 \sin A}$$

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\text{where } s = (a + b + c)/2$$

SPHERICAL TRIANGLES



Law of sines

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}$$

Law of cosines

$$\cos a = \cos b \cos c + \sin b \sin c \cos A$$

$$\text{Area of sphere} = 4\pi R^2$$

$$\text{Volume of sphere} = \frac{4}{3}\pi R^3$$

$$\text{Spherical excess in sec.} = \frac{bc \sin A}{9.7 \times 10^{-6} R^2}$$

where R = mean radius of the earth

PROBABILITY AND STATISTICS

$$\sigma = \sqrt{\frac{\sum(x_i - \bar{x})^2}{n-1}} = \sqrt{\frac{\sum v^2}{n-1}}$$

where:

σ = standard deviation (sometimes referred to as standard error)

$\sum v^2$ = sum of the squares of the residuals (deviation from the mean)

n = number of observations

\bar{x} = mean of the observations (individual measurements x_i)

$$\sigma_{\text{sum}} = \sqrt{\sigma_1^2 + \sigma_2^2 + \dots + \sigma_n^2}$$

$$\sigma_{\text{series}} = \sigma\sqrt{n}$$

$$\sigma_{\text{mean}} = \frac{\sigma}{\sqrt{n}}$$

$$\sigma_{\text{product}} = \sqrt{A^2 \sigma_b^2 + B^2 \sigma_a^2}$$

$$\Sigma = \begin{bmatrix} \sigma_x^2 & \sigma_{xy} \\ \sigma_{xy} & \sigma_y^2 \end{bmatrix}$$

$$\tan 2\theta = \frac{2\sigma_{xy}}{\sigma_x^2 - \sigma_y^2} \text{ where } \theta = \text{the counterclockwise angle from the x axis}$$

Relative weights are inversely proportional to variances, or:

$$W_a \propto \frac{1}{\sigma_a^2}$$

Weighted mean:

$$\bar{M}_w = \frac{\sum WM}{\sum W}$$

where:

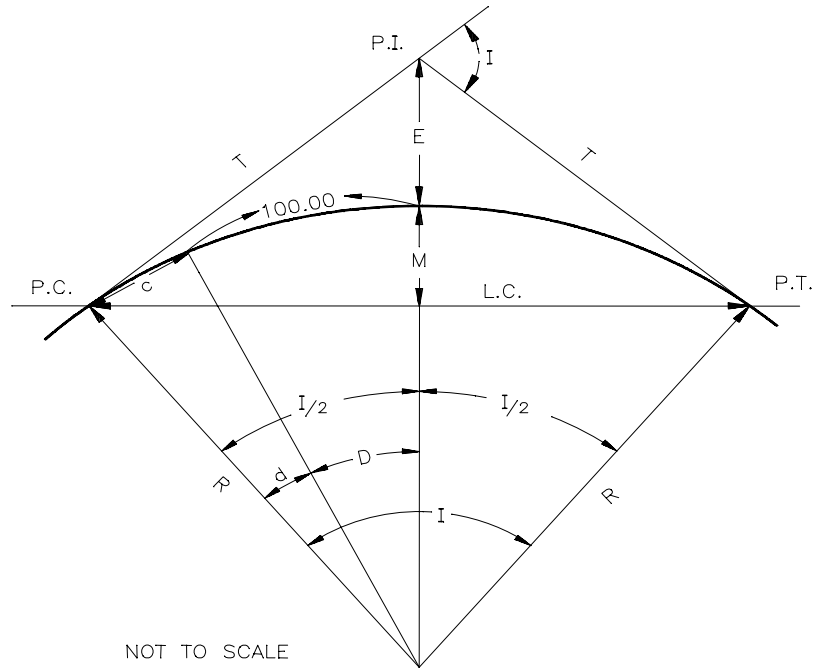
M_w = weighted mean

$\sum WM$ = sum of individual weights times their measurements

$\sum W$ = sum of the weights

HORIZONTAL CIRCULAR CURVES

- D = Degree of curve, arc definition
- L = Length of curve from P.C. to P.T.
- c = Length of sub-chord
- ℓ = Length of arc for sub-chord
- d = Central angle for sub-chord



$$D = \frac{5,729.58}{R}$$

$$T = R \tan(I/2)$$

$$L = RI \frac{\pi}{180} = \frac{I}{D}(100)$$

$$LC = 2R \sin(I/2)$$

$$c = 2R \sin(d/2)$$

$$d = \ell D / 100$$

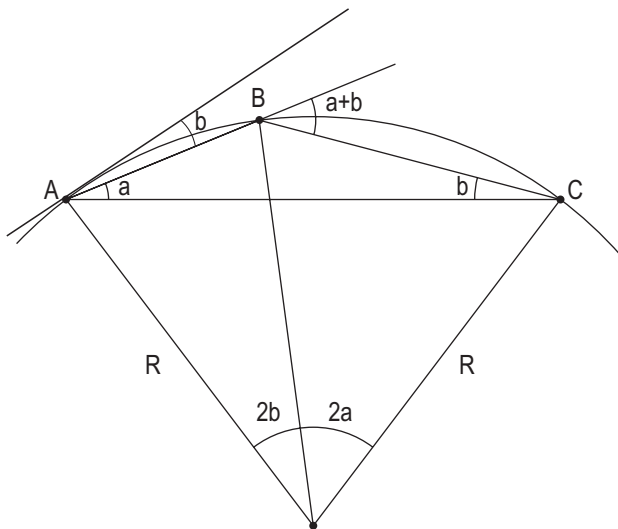
$$M = R [1 - \cos(I/2)]$$

$$E = R \left[\frac{1}{\cos(I/2)} - 1 \right]$$

$$\text{Area of sector} = \frac{RL}{2} = \frac{\pi R^2 I}{360}$$

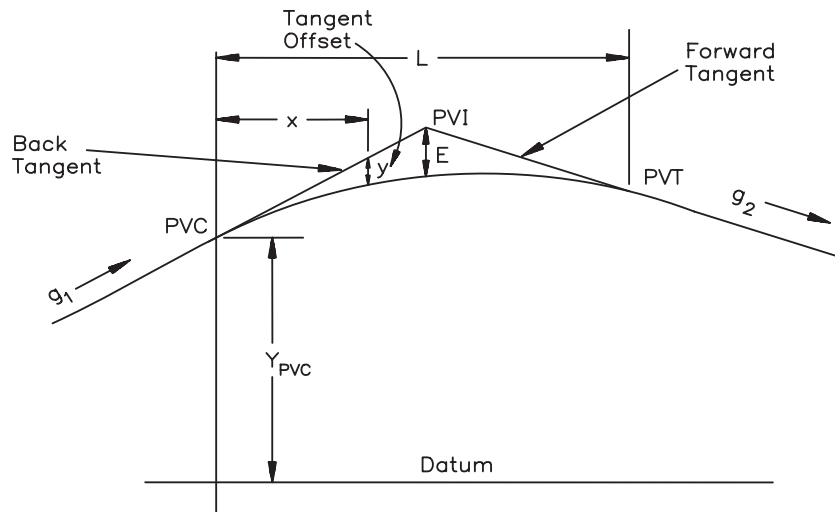
$$\text{Area of segment} = \frac{\pi R^2 I}{360} - \frac{R^2 \sin I}{2}$$

$$\text{Area between curve and tangents} = R(T - L/2)$$



$$R = \frac{AC}{2 \sin(a+b)}$$

VERTICAL CURVE FORMULAS



VERTICAL CURVE FORMULAS
NOT TO SCALE

L = Length of curve (horizontal)

PVC = Point of vertical curvature

PVI = Point of vertical intersection

PVT = Point of vertical tangency

g_1 = Grade of back tangent

g_2 = Grade of forward tangent

x = Horizontal distance from PVC
(or point of tangency) to point on curve

a = Parabola constant

y = Tangent offset

E = Tangent offset at PVI

r = Rate of change of grade

$$\begin{aligned} \text{Tangent elevation} &= Y_{PVC} + g_1x \\ \text{and} &= Y_{PVI} + g_2(x - L/2) \end{aligned}$$

$$\begin{aligned} \text{Curve elevation} &= Y_{PVC} + g_1x + ax^2 \\ &= Y_{PVC} + g_1x + [(g_2 - g_1)/(2L)]x^2 \end{aligned}$$

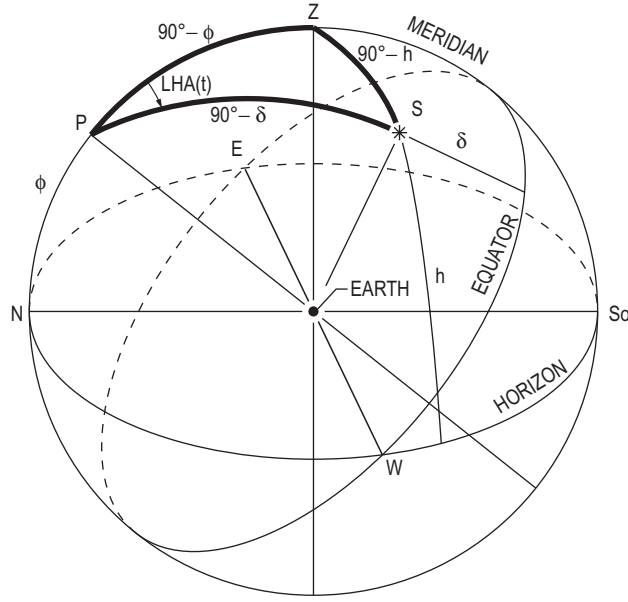
$$y = ax^2; \quad a = \frac{g_2 - g_1}{2L};$$

$$E = a \left(\frac{L}{2} \right)^2; \quad r = \frac{g_2 - g_1}{L}$$

Horizontal distance to min/max elevation on

$$\text{curve, } x_m = -\frac{g_1}{2a} = \frac{g_1 L}{g_1 - g_2}$$

ASTRONOMY



$$\cos(Az) = (\sin \delta - \sin \phi \sin h) / (\cos \phi \cos h) \quad (\text{altitude method})$$

$$\tan(Az) = -\sin(LHA) / (\cos \phi \tan \delta - \sin \phi \cos LHA) \quad (\text{hour angle method})$$

$$\sin h = \sin \phi \sin \delta + \cos \phi \cos \delta \cos LHA$$

$$t = LHA \text{ or } 360^\circ - LHA$$

$$\text{Horizontal circle correction for sun's semi-diameter} = SD / \cos h$$

Equations accurate for Polaris only:

$$h = \phi + p \cos LHA$$

$$Az = -(p \sin LHA) / \cos h$$

where:

Az = Azimuth (from north) to sun/star

δ = Declination

φ = Latitude

h = Altitude of sun/star

LHA = Local hour angle (sometimes referred to as "t" or "hour angle")

SD = Arc length of sun's semi-diameter

p = Polar distance of Polaris

PHOTOGRAMMETRY

$$\text{Scale} = \frac{ab}{AB} = \frac{f}{H-h}$$

(vertical photograph)

$$\text{Relief displacement} = \frac{rh}{H}$$

(vertical photograph)

Parallax equations:

$$p = x - x'$$

$$X = \frac{xB}{p}$$

$$Y = \frac{yB}{p}$$

$$h = H - \frac{fB}{p}$$

$$h_2 = h_1 + \frac{(p_2 - p_1)}{p_2} (H - h_1)$$

where:

- f = Focal length
- h = Height above datum
- H = Flying height above datum
- r = Radial distance from principal point
- p = Parallax measured on stereo pair
- B = Airbase of stereo pair
- x, y = Coordinates measured on left photo
- x' = Coordinate measured on right photo
- X, Y = Ground coordinates

PHYSICS

Lens equation:

$$\frac{1}{o} + \frac{1}{i} = \frac{1}{f}$$

where:

- o = Object distance
- i = Image distance
- f = Focal length

Snell laws:

$$n \sin \phi = n' \sin \phi'$$

where:

- n = Refractive index
- ϕ = Angle of incidence

Curvature and refraction:

$$(c + r) = 0.0206M^2$$

where:

- (c + r) = Combined effect of curvature and refraction in feet
- M = Distance in thousands of feet

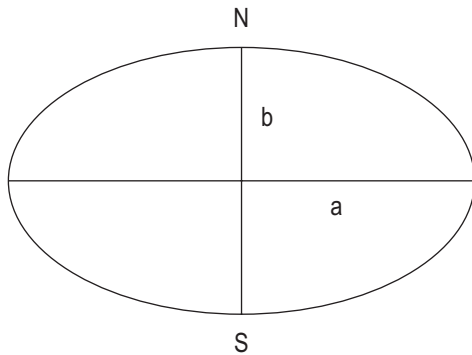
$$s = \frac{1}{2}at^2$$

where:

- s = Distance traveled starting from zero velocity
- a = Constant acceleration
- t = Time of travel

GEODESY

Ellipsoid



a = semi-major axis

b = semi-minor axis

$$\text{Flattening, } f = \frac{a - b}{a}$$

(usually published as $1/f$)

$$\text{Eccentricity, } e^2 = \frac{a^2 - b^2}{a^2}$$

$$\text{Radius in meridian, } M = \frac{a(1 - e^2)}{(1 - e^2 \sin^2 \phi)^{3/2}}$$

$$\text{Radius in prime vertical, } N = \frac{a}{(1 - e^2 \sin^2 \phi)^{1/2}}$$

Angular convergence of meridians

$$\theta_{\text{rad}} = \frac{d \tan \phi (1 - e^2 \sin^2 \phi)^{1/2}}{a}$$

Linear convergence of meridians

$$= \frac{\ell d \tan (1 - e^2 \sin^2 \phi)^{1/2}}{a}$$

where:

ϕ = Latitude

d = Distance along parallel at latitude ϕ

ℓ = Length along meridians separated by d

Ellipsoid definitions:

$$\text{GRS80: } a = 6,378,137.0 \text{ m}$$

$$1/f = 298.25722101$$

$$\text{Clark 1866: } a = 6,378,206.4 \text{ m}$$

$$1/f = 294.97869821$$

Orthometric correction:

$$\text{Correction} = -0.005288 \sin 2\phi h \Delta\phi \text{arc} 1'$$

where: ϕ = latitude at starting point

h = datum elevation in meters or feet
at starting point

$\Delta\phi$ = change in latitude in minutes
between the two points (+ in the
direction of increasing latitude or
towards the pole)

STATE PLANE COORDINATES

Scale factor = Grid distance/geodetic
(ellipsoidal) distance

Elevation factor = $R/(R + H + N)$

where:

R = Ellipsoid radius

H = Orthometric height

N = Geoid height

For precision less than $1/200,000$:

R = 20,906,000 ft

H = Elevation above sea level

N = 0

ELECTRONIC DISTANCE MEASUREMENT

$$V = c/n$$

$$\lambda = V/f$$

$$D = \frac{(m\lambda + d)}{2}$$

where:

- V = Velocity of light through the atmosphere (m/s)
- c = Velocity of light in a vacuum
- n = Index of refraction
- λ = Wave length (m)
- f = Modulated frequency in hertz (cycles/sec)
- D = Distance measured
- m = Integer number of full wavelengths
- d = Fractional part of the wavelength

ATMOSPHERIC CORRECTION

A 10°C temperature change or a pressure difference of 1 in. of mercury produces a distance correction of approximately 10 parts per million (ppm).

AREA FORMULAS

Area by coordinates where i is point order in a closed polygon.

$$\text{Area} = \frac{1}{2} \left[\sum_{i=1}^n X_i Y_{i+1} - \sum_{i=1}^n X_i Y_{i-1} \right]$$

Trapezoidal Rule

$$\text{Area} = w \left(\frac{h_1 + h_n}{2} + h_2 + h_3 + h_4 + \dots + h_{n-1} \right)$$

Simpson's 1/3 Rule

$$\text{Area} = w \left[h_1 + 2(\sum h_{\text{odds}}) + 4(\sum h_{\text{evens}}) + h_n \right] / 3$$

EARTHWORK FORMULAS

Average end area formula

$$\text{volume} = L(A_1 + A_2)/2$$

Prismoidal formula

$$\text{volume} = L(A_1 + 4A_m + A_2)/6$$

Pyramid or cone

$$\text{volume} = h(\text{Area of Base})/3$$

TAPE CORRECTION FORMULAS

Correction for temperature

$$C_t = 6.5 \times 10^{-6} (T - T_s)L$$

Correction for tension

$$C_p = (P - P_s)L/(AE)$$

Correction for sag

$$C_s = (w^2 l^3)/(24P^2)$$

where:

T = Temperature of tape during measurement, °F

T_s = Temperature of tape during calibration, °F

L = Distance measured, ft

P = Pull applied during measurement, lb

P_s = Pull applied during calibration, lb

A = Cross-sectional area of tape, in²

E = Modulus of elasticity of tape, psi

w = Weight of tape, lb/ft

l = Length of unsupported span, ft

STADIA

Horizontal distance = $KS \cos^2 \alpha$

Vertical distance = $KS \sin \alpha \cos \alpha$

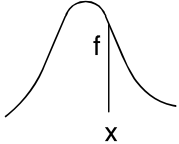
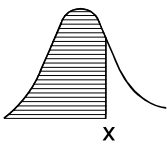
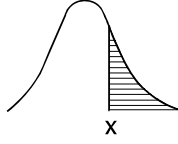
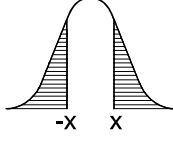
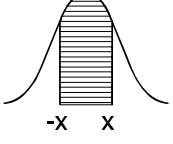
where:

K = Stadia interval factor (usually 100)

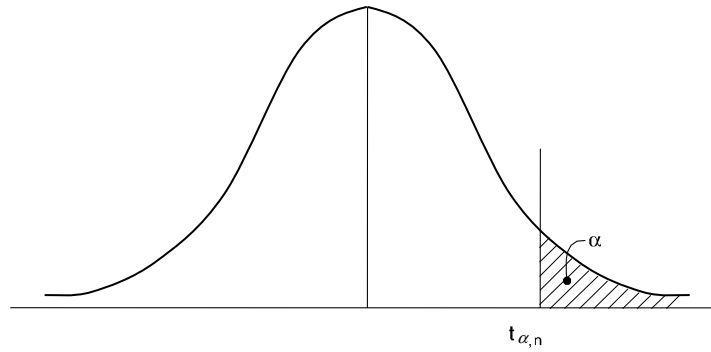
S = Rod intercept

α = Slope angle measured from horizontal

UNIT NORMAL DISTRIBUTION TABLE

| |  |  |  |  |  |
|-----------|---|---|---|---|---|
| x | f(x) | F(x) | R(x) | 2R(x) | W(x) |
| 0.0 | 0.3989 | 0.5000 | 0.5000 | 1.0000 | 0.0000 |
| 0.1 | 0.3970 | 0.5398 | 0.4602 | 0.9203 | 0.0797 |
| 0.2 | 0.3910 | 0.5793 | 0.4207 | 0.8415 | 0.1585 |
| 0.3 | 0.3814 | 0.6179 | 0.3821 | 0.7642 | 0.2358 |
| 0.4 | 0.3683 | 0.6554 | 0.3446 | 0.6892 | 0.3108 |
| 0.5 | 0.3521 | 0.6915 | 0.3085 | 0.6171 | 0.3829 |
| 0.6 | 0.3332 | 0.7257 | 0.2743 | 0.5485 | 0.4515 |
| 0.7 | 0.3123 | 0.7580 | 0.2420 | 0.4839 | 0.5161 |
| 0.8 | 0.2897 | 0.7881 | 0.2119 | 0.4237 | 0.5763 |
| 0.9 | 0.2661 | 0.8159 | 0.1841 | 0.3681 | 0.6319 |
| 1.0 | 0.2420 | 0.8413 | 0.1587 | 0.3173 | 0.6827 |
| 1.1 | 0.2179 | 0.8643 | 0.1357 | 0.2713 | 0.7287 |
| 1.2 | 0.1942 | 0.8849 | 0.1151 | 0.2301 | 0.7699 |
| 1.3 | 0.1714 | 0.9032 | 0.0968 | 0.1936 | 0.8064 |
| 1.4 | 0.1497 | 0.9192 | 0.0808 | 0.1615 | 0.8385 |
| 1.5 | 0.1295 | 0.9332 | 0.0668 | 0.1336 | 0.8664 |
| 1.6 | 0.1109 | 0.9452 | 0.0548 | 0.1096 | 0.8904 |
| 1.7 | 0.0940 | 0.9554 | 0.0446 | 0.0891 | 0.9109 |
| 1.8 | 0.0790 | 0.9641 | 0.0359 | 0.0719 | 0.9281 |
| 1.9 | 0.0656 | 0.9713 | 0.0287 | 0.0574 | 0.9426 |
| 2.0 | 0.0540 | 0.9772 | 0.0228 | 0.0455 | 0.9545 |
| 2.1 | 0.0440 | 0.9821 | 0.0179 | 0.0357 | 0.9643 |
| 2.2 | 0.0355 | 0.9861 | 0.0139 | 0.0278 | 0.9722 |
| 2.3 | 0.0283 | 0.9893 | 0.0107 | 0.0214 | 0.9786 |
| 2.4 | 0.0224 | 0.9918 | 0.0082 | 0.0164 | 0.9836 |
| 2.5 | 0.0175 | 0.9938 | 0.0062 | 0.0124 | 0.9876 |
| 2.6 | 0.0136 | 0.9953 | 0.0047 | 0.0093 | 0.9907 |
| 2.7 | 0.0104 | 0.9965 | 0.0035 | 0.0069 | 0.9931 |
| 2.8 | 0.0079 | 0.9974 | 0.0026 | 0.0051 | 0.9949 |
| 2.9 | 0.0060 | 0.9981 | 0.0019 | 0.0037 | 0.9963 |
| 3.0 | 0.0044 | 0.9987 | 0.0013 | 0.0027 | 0.9973 |
| Fractiles | | | | | |
| 1.2816 | 0.1755 | 0.9000 | 0.1000 | 0.2000 | 0.8000 |
| 1.6449 | 0.1031 | 0.9500 | 0.0500 | 0.1000 | 0.9000 |
| 1.9600 | 0.0584 | 0.9750 | 0.0250 | 0.0500 | 0.9500 |
| 2.0537 | 0.0484 | 0.9800 | 0.0200 | 0.0400 | 0.9600 |
| 2.3263 | 0.0267 | 0.9900 | 0.0100 | 0.0200 | 0.9800 |
| 2.5758 | 0.0145 | 0.9950 | 0.0050 | 0.0100 | 0.9900 |

t-DISTRIBUTION TABLE

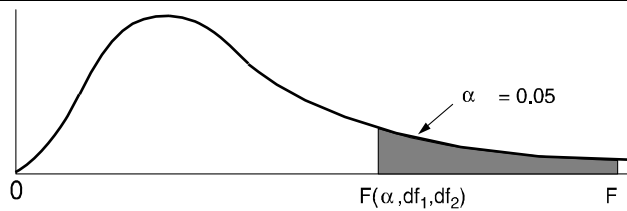


VALUES OF $t_{\alpha,n}$

| n | $\alpha = 0.10$ | $\alpha = 0.05$ | $\alpha = 0.025$ | $\alpha = 0.01$ | $\alpha = 0.005$ | n |
|----|-----------------|-----------------|------------------|-----------------|------------------|----|
| 1 | 3.078 | 6.314 | 12.706 | 31.821 | 63.657 | 1 |
| 2 | 1.886 | 2.920 | 4.303 | 6.965 | 9.925 | 2 |
| 3 | 1.638 | 2.353 | 3.182 | 4.541 | 5.841 | 3 |
| 4 | 1.533 | 2.132 | 2.776 | 3.747 | 4.604 | 4 |
| 5 | 1.476 | 2.015 | 2.571 | 3.365 | 4.032 | 5 |
| 6 | 1.440 | 1.943 | 2.447 | 3.143 | 3.707 | 6 |
| 7 | 1.415 | 1.895 | 2.365 | 2.998 | 3.499 | 7 |
| 8 | 1.397 | 1.860 | 2.306 | 2.896 | 3.355 | 8 |
| 9 | 1.383 | 1.833 | 2.262 | 2.821 | 3.250 | 9 |
| 10 | 1.372 | 1.812 | 2.228 | 2.764 | 3.169 | 10 |
| 11 | 1.363 | 1.796 | 2.201 | 2.718 | 3.106 | 11 |
| 12 | 1.356 | 1.782 | 2.179 | 2.681 | 3.055 | 12 |
| 13 | 1.350 | 1.771 | 2.160 | 2.650 | 3.012 | 13 |
| 14 | 1.345 | 1.761 | 2.145 | 2.624 | 2.977 | 14 |
| 15 | 1.341 | 1.753 | 2.131 | 2.602 | 2.947 | 15 |
| 16 | 1.337 | 1.746 | 2.120 | 2.583 | 2.921 | 16 |
| 17 | 1.333 | 1.740 | 2.110 | 2.567 | 2.898 | 17 |
| 18 | 1.330 | 1.734 | 2.101 | 2.552 | 2.878 | 18 |
| 19 | 1.328 | 1.729 | 2.093 | 2.539 | 2.861 | 19 |
| 20 | 1.325 | 1.725 | 2.086 | 2.528 | 2.845 | 20 |
| 21 | 1.323 | 1.721 | 2.080 | 2.518 | 2.831 | 21 |
| 22 | 1.321 | 1.717 | 2.074 | 2.508 | 2.819 | 22 |
| 23 | 1.319 | 1.714 | 2.069 | 2.500 | 2.807 | 23 |
| 24 | 1.318 | 1.711 | 2.064 | 2.492 | 2.797 | 24 |
| 25 | 1.316 | 1.708 | 2.060 | 2.485 | 2.787 | 25 |
| 26 | 1.315 | 1.706 | 2.056 | 2.479 | 2.779 | 26 |
| 27 | 1.314 | 1.703 | 2.052 | 2.473 | 2.771 | 27 |
| 28 | 1.313 | 1.701 | 2.048 | 2.467 | 2.763 | 28 |
| 29 | 1.311 | 1.699 | 2.045 | 2.462 | 2.756 | 29 |
| ∞ | 1.282 | 1.645 | 1.960 | 2.326 | 2.576 | ∞ |

CRITICAL VALUES OF THE F DISTRIBUTION – TABLE

For a particular combination of numerator and denominator degrees of freedom, entry represents the critical values of F corresponding to a specified upper tail area (α).



| Denominator df ₂ | Numerator df ₁ | | | | | | | | | | | | | | | | | | |
|--------------------------------|---------------------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 12 | 15 | 20 | 24 | 30 | 40 | 60 | 120 | ∞ |
| 1 | 161.4 | 199.5 | 215.7 | 224.6 | 230.2 | 234.0 | 236.8 | 238.9 | 240.5 | 241.9 | 243.9 | 245.9 | 248.0 | 249.1 | 250.1 | 251.1 | 252.2 | 253.3 | 254.3 |
| 2 | 18.51 | 19.00 | 19.16 | 19.25 | 19.30 | 19.33 | 19.35 | 19.37 | 19.38 | 19.40 | 19.41 | 19.43 | 19.45 | 19.45 | 19.46 | 19.47 | 19.48 | 19.49 | 19.50 |
| 3 | 10.13 | 9.55 | 9.28 | 9.12 | 9.01 | 8.94 | 8.89 | 8.85 | 8.81 | 8.79 | 8.74 | 8.70 | 8.66 | 8.64 | 8.62 | 8.59 | 8.57 | 8.55 | 8.53 |
| 4 | 7.71 | 6.94 | 6.59 | 6.39 | 6.26 | 6.16 | 6.09 | 6.04 | 6.00 | 5.96 | 5.91 | 5.86 | 5.80 | 5.77 | 5.75 | 5.72 | 5.69 | 5.66 | 5.63 |
| 5 | 6.61 | 5.79 | 5.41 | 5.19 | 5.05 | 4.95 | 4.88 | 4.82 | 4.77 | 4.74 | 4.68 | 4.62 | 4.56 | 4.53 | 4.50 | 4.46 | 4.43 | 4.40 | 4.36 |
| 6 | 5.99 | 5.14 | 4.76 | 4.53 | 4.39 | 4.28 | 4.21 | 4.15 | 4.10 | 4.06 | 4.00 | 3.94 | 3.87 | 3.84 | 3.81 | 3.77 | 3.74 | 3.70 | 3.67 |
| 7 | 5.59 | 4.74 | 4.35 | 4.12 | 3.97 | 3.87 | 3.79 | 3.73 | 3.68 | 3.64 | 3.57 | 3.51 | 3.44 | 3.41 | 3.38 | 3.34 | 3.30 | 3.27 | 3.23 |
| 8 | 5.32 | 4.46 | 4.07 | 3.84 | 3.69 | 3.58 | 3.50 | 3.44 | 3.39 | 3.35 | 3.28 | 3.22 | 3.15 | 3.12 | 3.08 | 3.04 | 3.01 | 2.97 | 2.93 |
| 9 | 5.12 | 4.26 | 3.86 | 3.63 | 3.48 | 3.37 | 3.29 | 3.23 | 3.18 | 3.14 | 3.07 | 3.01 | 2.94 | 2.90 | 2.86 | 2.83 | 2.79 | 2.75 | 2.71 |
| 10 | 4.96 | 4.10 | 3.71 | 3.48 | 3.33 | 3.22 | 3.14 | 3.07 | 3.02 | 2.98 | 2.91 | 2.85 | 2.77 | 2.74 | 2.70 | 2.66 | 2.62 | 2.58 | 2.54 |
| 11 | 4.84 | 3.98 | 3.59 | 3.36 | 3.20 | 3.09 | 3.01 | 2.95 | 2.90 | 2.85 | 2.79 | 2.72 | 2.65 | 2.61 | 2.57 | 2.53 | 2.49 | 2.45 | 2.40 |
| 12 | 4.75 | 3.89 | 3.49 | 3.26 | 3.11 | 3.00 | 2.91 | 2.85 | 2.80 | 2.75 | 2.69 | 2.62 | 2.54 | 2.51 | 2.47 | 2.43 | 2.38 | 2.34 | 2.30 |
| 13 | 4.67 | 3.81 | 3.41 | 3.18 | 3.03 | 2.92 | 2.83 | 2.77 | 2.71 | 2.67 | 2.60 | 2.53 | 2.46 | 2.42 | 2.38 | 2.34 | 2.30 | 2.25 | 2.21 |
| 14 | 4.60 | 3.74 | 3.34 | 3.11 | 2.96 | 2.85 | 2.76 | 2.70 | 2.65 | 2.60 | 2.53 | 2.46 | 2.39 | 2.35 | 2.31 | 2.27 | 2.22 | 2.18 | 2.13 |
| 15 | 4.54 | 3.68 | 3.29 | 3.06 | 2.90 | 2.79 | 2.71 | 2.64 | 2.59 | 2.54 | 2.48 | 2.40 | 2.33 | 2.29 | 2.25 | 2.20 | 2.16 | 2.11 | 2.07 |
| 16 | 4.49 | 3.63 | 3.24 | 3.01 | 2.85 | 2.74 | 2.66 | 2.59 | 2.54 | 2.49 | 2.42 | 2.35 | 2.28 | 2.24 | 2.19 | 2.15 | 2.11 | 2.06 | 2.01 |
| 17 | 4.45 | 3.59 | 3.20 | 2.96 | 2.81 | 2.70 | 2.61 | 2.55 | 2.49 | 2.45 | 2.38 | 2.31 | 2.23 | 2.19 | 2.15 | 2.10 | 2.06 | 2.01 | 1.96 |
| 18 | 4.41 | 3.55 | 3.16 | 2.93 | 2.77 | 2.66 | 2.58 | 2.51 | 2.46 | 2.41 | 2.34 | 2.27 | 2.19 | 2.15 | 2.11 | 2.06 | 2.02 | 1.97 | 1.92 |
| 19 | 4.38 | 3.52 | 3.13 | 2.90 | 2.74 | 2.63 | 2.54 | 2.48 | 2.42 | 2.38 | 2.31 | 2.23 | 2.16 | 2.11 | 2.07 | 2.03 | 1.98 | 1.93 | 1.88 |
| 20 | 4.35 | 3.49 | 3.10 | 2.87 | 2.71 | 2.60 | 2.51 | 2.45 | 2.39 | 2.35 | 2.28 | 2.20 | 2.12 | 2.08 | 2.04 | 1.99 | 1.95 | 1.90 | 1.84 |
| 21 | 4.32 | 3.47 | 3.07 | 2.84 | 2.68 | 2.57 | 2.49 | 2.42 | 2.37 | 2.32 | 2.25 | 2.18 | 2.10 | 2.05 | 2.01 | 1.96 | 1.92 | 1.87 | 1.81 |
| 22 | 4.30 | 3.44 | 3.05 | 2.82 | 2.66 | 2.55 | 2.46 | 2.40 | 2.34 | 2.30 | 2.23 | 2.15 | 2.07 | 2.03 | 1.98 | 1.94 | 1.89 | 1.84 | 1.78 |
| 23 | 4.28 | 3.42 | 3.03 | 2.80 | 2.64 | 2.53 | 2.44 | 2.37 | 2.32 | 2.27 | 2.20 | 2.13 | 2.05 | 2.01 | 1.96 | 1.91 | 1.86 | 1.81 | 1.76 |
| 24 | 4.26 | 3.40 | 3.01 | 2.78 | 2.62 | 2.51 | 2.42 | 2.36 | 2.30 | 2.25 | 2.18 | 2.11 | 2.03 | 1.98 | 1.94 | 1.89 | 1.84 | 1.79 | 1.73 |
| 25 | 4.24 | 3.39 | 2.99 | 2.76 | 2.60 | 2.49 | 2.40 | 2.34 | 2.28 | 2.24 | 2.16 | 2.09 | 2.01 | 1.96 | 1.92 | 1.87 | 1.82 | 1.77 | 1.71 |
| 26 | 4.23 | 3.37 | 2.98 | 2.74 | 2.59 | 2.47 | 2.39 | 2.32 | 2.27 | 2.22 | 2.15 | 2.07 | 1.99 | 1.95 | 1.90 | 1.85 | 1.80 | 1.75 | 1.69 |
| 27 | 4.21 | 3.35 | 2.96 | 2.73 | 2.57 | 2.46 | 2.37 | 2.31 | 2.25 | 2.20 | 2.13 | 2.06 | 1.97 | 1.93 | 1.88 | 1.84 | 1.79 | 1.73 | 1.67 |
| 28 | 4.20 | 3.34 | 2.95 | 2.71 | 2.56 | 2.45 | 2.36 | 2.29 | 2.24 | 2.19 | 2.12 | 2.04 | 1.96 | 1.91 | 1.87 | 1.82 | 1.77 | 1.71 | 1.65 |
| 29 | 4.18 | 3.33 | 2.93 | 2.70 | 2.55 | 2.43 | 2.35 | 2.28 | 2.22 | 2.18 | 2.10 | 2.03 | 1.94 | 1.90 | 1.85 | 1.81 | 1.75 | 1.70 | 1.64 |
| 30 | 4.17 | 3.32 | 2.92 | 2.69 | 2.53 | 2.42 | 2.33 | 2.27 | 2.21 | 2.16 | 2.09 | 2.01 | 1.93 | 1.89 | 1.84 | 1.79 | 1.74 | 1.68 | 1.62 |
| 40 | 4.08 | 3.23 | 2.84 | 2.61 | 2.45 | 2.34 | 2.25 | 2.18 | 2.12 | 2.08 | 2.00 | 1.92 | 1.84 | 1.79 | 1.74 | 1.69 | 1.64 | 1.58 | 1.51 |
| 60 | 4.00 | 3.15 | 2.76 | 2.53 | 2.37 | 2.25 | 2.17 | 2.10 | 2.04 | 1.99 | 1.92 | 1.84 | 1.75 | 1.70 | 1.65 | 1.59 | 1.53 | 1.47 | 1.39 |
| 120 | 3.92 | 3.07 | 2.68 | 2.45 | 2.29 | 2.17 | 2.09 | 2.02 | 1.96 | 1.91 | 1.83 | 1.75 | 1.66 | 1.61 | 1.55 | 1.50 | 1.43 | 1.35 | 1.25 |
| ∞ | 3.84 | 3.00 | 2.60 | 2.37 | 2.21 | 2.10 | 2.01 | 1.94 | 1.88 | 1.83 | 1.75 | 1.67 | 1.57 | 1.52 | 1.46 | 1.39 | 1.32 | 1.22 | 1.00 |

ECONOMICS

| Factor Name | Converts | Symbol | Formula |
|---------------------------------|------------------|-----------------|---|
| Single Payment Compound Amount | to F given P | $(F/P, i\%, n)$ | $(1 + i)^n$ |
| Single Payment Present Worth | to P given F | $(P/F, i\%, n)$ | $(1 + i)^{-n}$ |
| Uniform Series Sinking Fund | to A given F | $(A/F, i\%, n)$ | $\frac{i}{(1+i)^n - 1}$ |
| Capital Recovery | to A given P | $(A/P, i\%, n)$ | $\frac{i(1+i)^n}{(1+i)^n - 1}$ |
| Uniform Series Compound Amount | to F given A | $(F/A, i\%, n)$ | $\frac{(1+i)^n - 1}{i}$ |
| Uniform Series Present Worth | to P given A | $(P/A, i\%, n)$ | $\frac{(1+i)^n - 1}{i(1+i)^n}$ |
| Uniform Gradient Present Worth | to P given G | $(P/G, i\%, n)$ | $\frac{(1+i)^n - 1}{i^2(1+i)^n} - \frac{n}{i(1+i)^n}$ |
| Uniform Gradient † Future Worth | to F given G | $(F/G, i\%, n)$ | $\frac{(1+i)^n - 1}{i^2} - \frac{n}{i}$ |
| Uniform Gradient Uniform Series | to A given G | $(A/G, i\%, n)$ | $\frac{1}{i} - \frac{n}{(1+i)^n - 1}$ |

Nomenclature and Definitions

| | |
|-------|---|
| A | Uniform amount per interest period |
| B | Benefit |
| BV | Book Value |
| C | Cost |
| d | Combined interest rate per interest period |
| D_j | Depreciation in year j |
| F | Future worth, value, or amount |
| f | General inflation rate per interest period |
| G | Uniform gradient amount per interest period |
| i | Interest rate per interest period |
| i_e | Annual effective interest rate |
| m | Number of compounding periods per year |
| n | Number of compounding periods; or the expected life of an asset |
| P | Present worth, value, or amount |
| r | Nominal annual interest rate |
| S_n | Expected salvage value in year n |

Subscripts

| | |
|-----|--|
| j | at time j |
| n | at time n |
| † | $F/G = (F/A - n)/i = (F/A) \times (A/G)$ |

Nonannual Compounding

$$i_e = \left(1 + \frac{r}{m}\right)^m - 1$$

Book Value

$$BV = \text{Initial cost} - \sum D_j$$

Depreciation

$$\text{Straight line } D_j = \frac{C - S_n}{n}$$

Accelerated Cost Recovery System (ACRS)

$$D_j = (\text{factor from table below}) C$$

| MODIFIED ACRS FACTORS | | | | |
|------------------------------|--------------------------------|-------------|------------|------------|
| Year | Recovery Period (Years) | | | |
| | 3 | 5 | 7 | 10 |
| | Recovery Rate (%) | | | |
| 1 | 33.3 | 20.0 | 14.3 | 10.0 |
| 2 | 44.5 | 32.0 | 24.5 | 18.0 |
| 3 | 14.8 | 19.2 | 17.5 | 14.4 |
| 4 | 7.4 | 11.5 | 12.5 | 11.5 |
| 5 | | 11.5 | 8.9 | 9.2 |
| 6 | | 5.8 | 8.9 | 7.4 |
| 7 | | | 8.9 | 6.6 |
| 8 | | | 4.5 | 6.6 |
| 9 | | | | 6.5 |
| 10 | | | | 6.5 |
| 11 | | | | 3.3 |

Capitalized Costs

Capitalized costs are present worth values using an assumed perpetual period of time.

$$\text{Capitalized costs} = P = \frac{A}{i}$$